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Latent Profiles of Korean Preschool Teachers' Three Facets of Pedagogical Content Knowledge in Early Mathematics

Im, Haesung[1)]
Duksung Women's University

Choi, Jinkyung
Hansung University

Abstract

Existing research primarily focused on only one aspect of preschool teachers' pedagogical content knowledge (PCK) in early mathematics. With a person-centered approach, this study identified profiles of preschool teachers by three facets of PCK, which include (1) content knowledge in mathematics within teacher-directed instruction, (2) teachers' sensitivity to mathematics during children's play, and (3) the complexity of math questions. A total of 148 Korean classroom teachers nested in 40 preschools were provided with a play-like scenario where they need to recognize mathematical concepts and effective questioning strategies during children's play. Findings from Latent Profile Analysis indicated that the three facets of PCK in early mathematics are distinct sets of knowledge and skills that are qualitatively different. This finding calls for more intervention research on supporting Korean preschool teachers' multifaceted PCK in early mathematics.

***Keywords**:* Pedagogical content knowledge in early mathematics, sensitivity to mathematics during children's play, the complexity of math questions, Latent Profile Analysis (LPA)

Corresponding author, [1)] delightlord37@duksung.ac.kr

Introduction

Recently, early childhood mathematics education has been at the center of attention for educators, policymakers, and researchers internationally (Presser et al., 2015). International research provided compelling evidence that while early literacy skills predict only later reading ability (Duncan & Magnuson, 2011), early mathematical skills were found to be more predictive of their later mathematical and reading skills than early literacy skills were (Cross et al., 2009; Nguyen et al., 2016). Additionally, these studies have shown that high-quality mathematical education predicts children's learning gains during the early years (Duncan & Magnuson, 2011).

Teachers' competencies for teaching have been recognized as one of the critical elements for high-quality mathematics education (Pelkowski et al., 2019; Trawick-Smith et al., 2016). As one of these competencies, pedagogical content knowledge (PCK) of mathematics includes subject knowledge in mathematics and how to teach it to young children. Initially, the term 'PCK' was coined by educational psychologist Lee Shulman in 1987. PCK can be defined as "the blending of content and pedagogy into an understanding of how particular topics are organized, [and] represented" (Shulman, 1987, p. 4). The plethora of research evidence indicated that teachers' content knowledge in mathematics correlates with high-quality math instruction, teacher's efficacy, students' positive attitude toward mathematics, and mathematic achievement (Oppermann et al., 2016; Zhang, 2015).

PCK in early mathematics is multifaceted and complex. It provides teachers with skills and knowledge from a range of teacher-directed to child-led activities (Epstein, 2014). However, the existing studies tended to focus on only one aspect of PCK in early mathematics (Björklund et al., 2018). While one line of research focused on teachers' content knowledge for teacher-directed math instruction (Hong & Chung, 2013; Kim & Hong, 2015; Lee, 2010), other lines of research focused on teachers' content knowledge for play-based learning (Trawick-Smith et al., 2016; Vogt et al., 2018). These differences drew from a contrasting pedagogical stance in teaching early mathematics (Björklund et al., 2018; Lee, 2017; Oppermann et al., 2016). There has been an ongoing debate about whether children should be learning mathematics through teacher-directed instruction or play-based learning (Figueiredo et al., 2018). Thus, we aim to examine the profiles of teachers by three facets of teachers' PCK in early mathematics: (1) content knowledge in mathematics within teacher-directed instruction, (2) teachers' sensitivity to mathematics during children's play, and (3) the complexity of teachers' math questions.

First, most of the previous studies were centered around teachers' content knowledge for direct math instruction (Hong & Chung, 2013; Kim & Hong, 2015; Lee, 2010). According to Shulman (1987), PCK in early mathematics can be defined as a set of subject matter knowledge for teaching mathematics to young children. Teachers with a high-level of content knowledge can plan specific mathematic activities to achieve learning goals (Hill et al., 2008). These teachers are competent in organizing mathematical activities and providing mathematical explanations (Lee, 2017; Kim & Connelly, 2019). However, this approach failed to capture the distinctive nature of teaching young children through play-based learning (Kim & Seo, 2019; Lee, 2017; McCray & Chen, 2012).

Second, another line of researchers highlighted the importance of teachers' content knowledge for facilitating mathematic learning during children's play (Kim & Park, 2017; Trawick-Smith et al., 2016; Vogt et al., 2018). According to Oppermann et al. (2016), sensitivity to mathematics during children's play can be defined as the teachers' ability to recognize mathematical concepts during children's play. Unlike elementary school children, younger children learn through play, as they need to divide the blocks into equal amounts with their friends (Moon & Lee, 2011; Trawick-Smith et al., 2016). These critical instances where certain types of mathematical concepts are easy to learn are called "teachable moments." Therefore, teachers need to recognize children's emerging mathematical thinking within the context that appears in free play (Kim & Seo, 2019; Lee, 2011).

Third, teachers' complexity of questions has been identified as an additional critical facet of pedagogical knowledge in extending children's mathematical thinking during play (McCray & Chen, 2012). For example, teachers' cognitively challenging questions during play would help children to connect abstract ideas to real-world situations (Uscianowski et al., 2020). While some teachers may use questions that only lead to simple answers, other teachers may provide questions that scaffold children's existing knowledge (Deshmukh et al., 2019). Therefore, the complexity of questions can be evaluated by the level of children's emerging cognitive engagement. However, studies indicate that teachers generally have difficulties in posing complex math questions during play (Uscianowski et al., 2020).

The Current Study

As noted above, existing studies tended to be one-sided, focusing on opposite ends of the continuum on PCK, either for teacher-directed instruction or play-based learning (Björklund et al., 2018; Lewis et al., 2019). Moreover, previous studies that assume teachers' content knowledge is the prerequisite for teachers' pedagogical knowledge, without empirical evidence (Cross et al., 2009). Despite the intuitive appeal in the link between teachers' content knowledge and pedagogical knowledge, there was a lack of research that provided empirical evidence. Moreover, existing studies have primarily employed a variable-centered approach, which is not suited for explicating inter-related relationships among three facets of PCK in early mathematics. Therefore, more studies are needed to investigate the inter-relationships amongst three facets of PCK in early mathematics with a person-centered approach.

In line with the debate over teacher-directed math instruction and play-based learning, South Korea provides an exciting example of a tension between national play-based curriculum and teachers' implicit pedagogical stance in teacher-directed learning (Choi, 2019; Jung, 2013). Although the Korean national curriculum was influenced by Western child-centered and play-based theories, Korean teachers' classroom practices reflect Korean teachers' implicit folk pedagogy in teacher-directed learning (Ahn, 2015; Choi, 2019). Studies indicated that while Korean teachers typically advocate for the pedagogy of learning through play, their classroom practice is more aligned with teacher-directed instruction (Ahn, 2015).

Studies have also indicated that many Korean preschool teachers have difficulties in facilitating mathematical learning through play (Han & Park, 2016; Jung, 2013; Lee & Park, 2019). Studies indicate that early childhood teachers typically do not take sufficient time to expand the mathematical thinking of young children during free play (Lee & Lee, 2010). Moreover, the idea of capitalizing on teachable moments for mathematical education during children's play is a relatively new concept in South Korea (Kim & Park, 2017; Lee, 2017). Thus, more studies are needed to investigate Korean teachers' PCK in early mathematics.

As such, the present study aims to investigate three facets of PCK in early mathematics with a person-centered approach: (1) content knowledge in mathematics (2) sensitivity to mathematics during children's play, and (3) teachers' complexity of math questions. With a Latent Profile Analysis, this study will shed light on identifying distinctive profiles of preschool teachers' PCK in early mathematics, and how they differ by teachers' characteristics (Hu et al., 2016). The specific research questions are as follows:

1. How are preschool teachers' content knowledge in early mathematics, sensitivity to mathematics during children's play, and the ability to pose complex math questions during children's play related?
2. What patterns emerge in early childhood teachers' PCK, sensitivity to mathematics during children's play, and teachers' complexity of math questions?
3. How do teachers' profiles differ by teacher characteristics (e.g., major, level of education, years of teaching experiences, and teachers' age)?

Method

Context and Participants

Korea has a national curriculum for early childhood education with an emphasis on a play-based, child-centered approach (Korea Institute of Child Care and Education, 2019). *The Revised Nuri National Preschool Curriculum (2019)*, the Korean national curriculum, provided a guideline for preschool teachers to teach mathematics, language, literacy, social studies, and art for young children since 1969. It has five content areas with learning objectives and general guiding principles for teaching young children. According to the Korean Ministry of Education (2019), the *Revised Nuri National Preschool Curriculum* prioritizes the immersion of child-centered, play-based learning over teacher-directed activities. Notably, the curriculum aims to extend and enrich children's learning experiences through child-initiated play, which highlights the importance of teachers' ability to capitalize on teachable moments during children's free play (Korea Institute of Child Care and Education, 2019).

In this study, we utilized the snowball sampling strategy, where existing study participants recruit future participants from among their acquaintances. A total of 148 preschool teachers in Seoul, South Korea, participated in this study. Table 1 displays participants' demographic information, including their organizational type, level of teacher education, and years of teaching experience.

Table 1. *Socio-Demographic Information of the Study Participants (N=147)*

Demographic Variable	Frequency (%)	
Organization Types		
Public Preschool	17	(11.50)
Private Preschool	32	(21.60)
Other Types of Preschool	9	(6.10)
Public Daycare	23	(15.50)
Workplace Daycare	60	(40.50)
Non-response	7	(4.70)
Level of Teacher Education		
College Degree	27	(18.20)
Bachelor of Education	82	(55.40)
Master of Education	28	(18.90)
Above Grade Degree	9	(6.10)
Non-response	2	(1.40)
Teaching Experience		
Less than Three Years	32	(21.80)
3 - 5 Years	48	(32.70)
5 -7 Years	28	(19.00)
7 Years Above	35	(23.80)
Non-response	4	(2.70)

As can be seen in Table 1, a total of 40.50% of participants were employed at a workplace daycare, while 21.60% were employed at a private preschool, and 11.50% of teachers were working at a public preschool. The majority of teachers had bachelor's degrees (55.40%), while some teachers had master's degrees (18.90%). A total of 21.80% of teachers had less than three years of teaching experience, and the majority of teachers had more than three years of teaching experience.

Measures

Preschool Teachers' Content Knowledge in Mathematics

Preschool teachers' content knowledge was assessed by the Survey of PCK in Early Childhood Mathematics (SPECKECM), which had been developed by Smith (2000). This measure assessed preschool teachers' content knowledge in a total of 15 items in six content areas: number sense, patterns, ordering, shape, spatial sense, and comparisons. We coded 1 for the correct answer, and 0 for the incorrect answer. As each of the six subcategories had more than one item, we used the total scores for Latent Profile Analysis.

Sensitivity to Math during Children's Play

Teachers' sensitivity to mathematics during children's play was assessed by an instrument developed and validated by McCray and Chen (2012). While this instrument was initially adapted from the Preschool Mathematics PCK interview (PM-PCK interview), Kim and Park (2017) revised the scenario for open-ended questions in a survey format. With the added adaptation, we were able to provide participants with a play-like scenario with pictures that helped teachers easily understand the situation. The scenario was as follows:

"Britta and Jacob are playing together with dolls. They want to put their five babies to bed. Since there are no doll beds, they construct them one out of three shoeboxes. Jacob says, "but there aren't enough cribs." Britta responds, "These babies are younger," picking out the three babies with no hair and setting them near the shoeboxes. She picks up the two babies with thick hair, says "these babies don't need to nap anymore," and sets them aside. Jacob says, "OK, but this baby needs the most room" and puts the biggest bald baby in the biggest shoebox. Britta watches him and then puts the medium-sized bald baby in the medium-sized shoebox and the smallest bald baby in the smallest shoebox. Jacob says, "Now go to sleep, babies."

In this scenario, teachers needed to recognize mathematical concepts in six content areas: numbers, operations, geometry, patterns, data, and measurement. For instance, "recognizing that there are not enough cribs requires one-to-one correspondence" (Oppermann et al., 2016, p. 177). We scored five points for those teachers who were able to identify the correct mathematical concepts in the given scenario. In contrast, we scored one point for incorrect mathematical concepts and zero points for non-response.

The Complexity of Math Questions

Adopted from Uscianowski et al. (2020), teachers' questions were divided into three levels based on the complexity of the math questions. We asked participants to write possible questions once they noticed mathematical concepts from the play scenario. For example, teachers were asked to write possible questions on how they might expand children's understanding of mathematical concepts during play (see Table 2). We scored zero points when the teacher could not come up with any questions. Level I questions did not lead children to recognize relevant mathematical concepts. Level II questions helped children to notice the given mathematical concepts. Level III questions pushed children to infer, predict, problem-solve, and formulate a solution, for a score of five points.

Table 2. *The Description and Examples of The Complexity of Math Questions*

Level	Description	Question Example (e.g., Measurement Concept)	Score
Level Ⅰ	This type of question does not help a child to recognize the 'correct' mathematical concepts in the given scenario	Why did Eunseo put away two big dolls?	1 point
Level Ⅱ	This type of question helps a child to recognize the correct mathematical concepts in the given scenario	What would be the size of a box this little doll need?	3 points
Level Ⅲ	This type of question helps a child to extend and scaffold children's mathematical thinking	In what ways can you compare the size of the dolls?	5 points

To ensure inter-rater reliability, two researchers coded the full data independently. Then, the researchers compared their scores to check whether it had reached the acceptable level of inter-rater consistency for each domain. After an iterative process, 95% of inter-coder reliability was established.

Analysis

With a person-centered approach, this study utilized advanced statistics using Latent Profile Analysis. One of the benefits of using a person-centered approach is "to allow examining [of] all of the inter-related dimensions at once" (Hu et al., 2016, p. 60). The mixture modeling strategy was employed to identify latent subgroups (profiles) that displayed similar patterns across the sensitivity to mathematics during children's play, using Mplus 7.0 (Muthén & Muthén, 2012). Latent Profile Analysis (LPA) was a statistical procedure that could be used to categorize individual teachers into homogeneous subgroups by classifying teachers with a set of scores on PCK in early mathematics. Moreover, we examined if there were any profile differences by teacher characteristics.

Results

Descriptive Statistics

Table 3 represents descriptive statistics for preschool teachers' three facets of PCK in early mathematics, respectively. The mean scores obtained from the subcategories of content knowledge in mathematics were ranked from the highest to lowest: pattern (M=2.82; SD=2.50), number (M=1.92, SD=.98), shape (M=1.64; SD=.59), comparison (M=1.40, SD=.57), and order (M=1.12, SD=.89).

Participants had the highest scores on identifying "number" concepts (M=4.01, SD=1.99), and "operation" concepts (M=3.67, SD=2.21) during children's play. However, participants generally had difficulties in noticing mathematical concepts of data analysis (M=1.42, SD=2.26) and space (M=1.54, SD=2.30). Similarly, participants were able to pose complex math questions on mathematical concepts of measurement (M=2.74, SD=1.87) and number

(M=2.65, SD=1.38). At the same time, they had difficulty in posing complex math questions on mathematical concepts of space (M=1.25, SD=1.82) and data analysis (M=1.35, SD=1.93). The bivariate correlations among key variables were presented in Table 4.

Table 3. *Descriptive Statistics of PCK in Early Mathematics*

	M	SD	Skewness	Kurtosis
Content Knowledge in Mathematics				
Number	1.95	.98	-.63	-.60
Shape	1.64	.59	-1.43	1.04
Oder	1.12	.89	.50	-.41
pattern	2.82	1.16	-.56	-.87
Comparison	1.40	.57	-.28	-.78
Total	8.84	2.54	-.33	.08
Sensitivity to Math during play				
Number	4.01	1.99	-1.54	.40
Operation	3.67	2.21	-1.07	-.86
Shape	2.40	2.50	.08	-2.02
Space	1.54	2.30	.85	-1.29
Pattern	2.33	2.50	.14	-2.01
Measurement	3.58	2.25	-.97	-1.07
Data Analysis	1.42	2.26	.96	-1.09
Total	18.55	9.82	-.36	-.68
The complexity of Math Questions				
Number	2.65	1.38	-.81	.37
Operation	2.60	1.64	-.42	-.71
Shape	2.42	2.17	-.02	-1.70
Space	1.25	1.82	.99	-.60
Pattern	1.62	1.95	.59	-1.25
Measurement	2.74	1.87	-.37	-1.14
Data Analysis	1.35	1.93	.91	-.82
Total	14.42	8.24	-.09	-.89

Note. N = 148

Table 4. *Bivariate Correlation Between Study Variables*

Variables	1	2	3	4	5	6	7
1. Content Knowledge in Mathematics	1						
2. Sensitivity to Math during play	.05	1					
3. The complexity of Math Questions	.12	.56***	1				
4. Major	.02	-.14	-.19*	1			
5. Level of Education	.06	-.14	-.12	.01	1		
6. Years of Teaching Experience	.08	.12	.12	-.09	.06	1	
7. Age of Teachers	-.04	-.29***	-.08	-.11	.31***	.09	1

Note. 1 = Content Knowledge in Mathematics, 2 = Sensitivity to Math during play, 3 = Complexity of Math Questions, 4 = Major, 5 = Level of Education (1 = Early Childhood Education, 2 =Child Care and Family Study, 3=Other Major), 6 = Years of Teaching Experience, 7 = Age of Teachers
**p* <.05. ***p*<.01. ****p*<.001.

Preschool teachers who had the sensitivity to mathematical concepts during children's play were more likely to ask complex math questions during children's play (r =.56, p<.000). However, there was no significant correlation between the content knowledge in mathematics and sensitivity to mathematics during children's play (r =.05, p > .05). Similarly, there was no significant correlation between the content knowledge in mathematics and the complexity of math questions (r =.12, p > .05). Teachers with early childhood education majors were more likely to pose complex math questions during children's play compared to preschool teachers with other majors (r =-.19, p<.05). Younger teachers had higher sensitivity in noticing mathematical concepts during children's play (r =.29, p <.001). However, the teacher's level of education was not significantly correlated with content knowledge in mathematics, and with sensitivity to math during play.

Latent Profile Analysis

We selected four Latent profile models as the best fitting model based on the recommendation by Peugh and Fan (2013). These criteria included information criteria, entropy, and the Likelihood Ratio Test. The enumeration indices of the competing five LPA models are presented in Table 5.

Table 5. *Latent Profile Analysis model-fit statistics*

Model	AIC	BIC	ABIC	ENTROPY	LMR-LR	ALMR LR	BLRT
1 profile	2845.31	2863.29	2844.30	-	-	-	-
2 profiles	2800.43	2830.40	2798.76	.69	.00	.00	.00
3 profiles	2789.41	2831.37	2787.07	.74	.06	.06	.01
4 profiles	**2781.37**	**2835.32**	**2778.36**	**.80**	**.05**	**.06**	**.02**
5 profiles	2780.49	2846.43	2776.81	.78	.29	.32	.31

Notes. AIC=Akaike's Information Criterion; BIC=Bayesian Information Criterion; ABIC=Adjusted Bayesian Information Criterion; ENTROPY = an indicator of how well the model classifies individuals into different latent profiles, with values closer to 1 indicating better classification; LMR=Lo-Mendell-Rubin test; ALMR LR= Adjusted Lo-Mendell-Rubin test BLTR=parametric bootstrapped likelihood ratio test, p-value.

As displayed in Table 5, the four-profile solution was the best fitting model for several reasons. First, the Bayesian Information Criterion (BIC) values continued to decrease, while the entropy of each model continued to increase by four-profile solutions. While a lower value on BIC indicated a better solution, an entropy greater than 0.80 was regarded as a high-quality classification (Tein et al., 2013). In this study, LMR-LR, ALMR LR, and BLRT, indicating either a two-profile solution or four-profile solution, fit the data well, as a significant *p*-value, less than .05 would indicate that there was no significant difference among identified profiles. However, we selected the four-profile solutions because it showed a lower BIC value with higher entropy.

Table 6. *Number of Membership by Different Latent Profile Analysis*

Model	Profile 1 n (%)	Profile 2 n (%)	Profile 3 n (%)	Profile 4 n (%)	Profile 5 n (%)
1 profile	148(100)				
2 profiles	68(45.95)	80(54.05)			
3 profiles	24(16.21)	70(47.30)	54(36.49)		
4 profiles	58(39.19)	63(42.57)	15(10.14)	12(8.10)	
5 profiles	15(10.14)	57(38.51)	12(8.11)	13(8.78)	51(34.46)

According to Geiser (2012), a profile with the smallest member needed to be over 5%. With the four-profile solution, the profile with the smallest member (profile 4) was 8.10% (see Figure 1). For model fit evaluation and model comparisons in LPA, statistical criteria played an important role. Given the interpretability and other considerations, we selected the four-profile model as the best solution (Collins & Lanza, 2009). Means of three facets of PCK in early mathematics are suggested in Table 7.

Table 7. *Means of Three Facets of PCK in Early Mathematics by 4 Profile Solutions*

	Profile 1	Profile 2	Profile 3	Profile 4	*F*
Content Knowledge in Mathematics	8.62	8.98	8.60	9.41	0.46
Sensitivity to Math during play	17.72	26.03	1.67	4.33	126.06***
Complexity of Math Questions	9.14	21.76	1.60	17.42	152.33***

**p* <.05. ***p* <.01. ****p* <.001

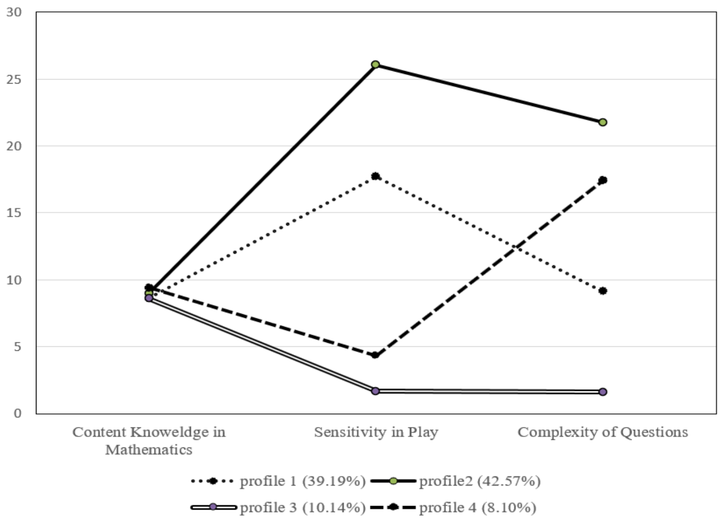

Figure 1. Four-profiles of PCK in Early Mathematics

As displayed in Table 7, there were no significant differences in the means of content knowledge in mathematics by four profiles (F = 0.46, p > 0.5). However, there were significant mean differences across four profiles by sensitivity to math during children's play ($F = 126.06$, $p < 0.01$), and by the complexity of math questions ($F = 152.33$, $p < 0.01$). Figure 1 describes the latent profiles of teachers by three facets of PCK in early mathematics.

Teacher Characteristics by Profiles

As noted earlier in Table 7, we provided descriptions of teachers' PCK in early mathematics by sensitivity to mathematics during children's play and the complexity of math questions. However, we did not describe further distinction on content knowledge in mathematics as there were no significant mean differences by profiles. Table 8 presents how four profiles differ by teacher characteristics, such as major, level of teacher education, years of teaching experience, and age of the teachers.

Table 8. *Teacher Characteristics by Profiles*

Teacher Characteristics	Profiles				Total Sample n (%)	*F*	*df*	φ
	1 39.19%	2 42.57%	3 10.14%	4 8.10%				
	n (%)	n (%)	n (%)	n (%)				
Major						23.04^{**}	6	.001
Early Childhood Education	24(42.10)	48(76.20)	8(53.30)	6(50.00)	86(58.50)			
Child Care and Family Study	32(56.10)	11(17.5)	6(40.00)	4(33.30)	53(36.10)			
Other Major	1(1.80)	4(6.30)	1(6.70)	2(16.70)	8(5.40)			
Level of Education						8.58	6	.20
College	5(35.70)	13(68.40)	1(33.30)	1(50.00)	20(52.60)			
Bachelor	7(50.00)	2(10.50)	2(66.70)	1(50.00)	12(31.60)			
Master Degree	2(14.30)	4(21.10)	0	0	6(15.80)			
Teaching Experience					`	9.59^{*}	3	.02
0-3 years	17(30.90)	30(48.40)	2(14.30)	2(16.70)	51(35.70)			
3 years above	38(69.10)	32(51.60)	12(85.70)	10(83.30)	92(64.30)			
Ages of Teachers						13.48^{*}	6	.03
20-30 years old	25(47.20)	30(65.20)	3(33.30)	2(16.70)	60(50.00)			
30-40 years old	25(47.20)	12(26.10)	4(44.40)	8(66.70)	49(40.80)			
40-50 years old	3(5.70)	4(8.70)	2(22.20)	2(16.70)	11(9.20)			

$^{*}p < .05$. $^{**}p < .01$. $^{***}p < .001$

Profile 1: Medium-level of PCK in early mathematics (n=58, prevalience= 39.19%)

This profile of teachers showed medium competencies compared to teachers of other profiles in this study. Teachers under this profile were able to recognize mathematical concepts during children's play to some degree (*M* = 17.72). These teachers came up with relevant questions that helped children to notice mathematical concepts (*M* = 9.14). However, they were not able to pose cognitively challenging questions. The majority of these teachers majored in child-care and family studies (56.10%). Less than half of these teachers majored in early childhood education (42.10%), while only a few teachers majored in non-early childhood related majors (1.80%). These teachers had a relatively high level of education compared to the total sample. For example, most of these teachers had either a bachelor's degree (50.00%) or above a master's degree (14.30%).

Profile 2: The Highest-level of PCK in early mathematics (n= 63, prevailence= 42.57%)

Teachers under the highest-profile showed the uppermost ability to recognize the relevant mathematical concepts during children's play (*M* = 26.03). Also, they came up with relatively complex math questions (*M* = 21.76). The majority of these teachers majored in early childhood education (76.20%). In contrast, only a few teachers majored in child-care and family studies (17.5%). However, teachers under this profile did not differ by years of teaching experience. Approximately half of these teachers were novice teachers (48.40%), and half of the remaining teachers were more experienced teachers (51.60%). The majority of teachers in profile 2 belonged to younger generations. For example, 65.20% of these teachers were in their twenties. Only a few teachers in this profile were between 40 to 50 years old (8.70%).

Profile 3: Low-level of sensitivity with high-level of complex math questions (n= 15, prevalence=10.14%)

Teachers under this profile had difficulties in noticing mathematical concepts during children's play (*M* = 4.33). However, they were able to come up with complex math questions (*M* =17.42). It is noteworthy that these teachers were able to pose more complex math questions compared to teachers under profile 2, who recognized relevant mathematical concepts in the given play-like scenario. Among these teachers, half majored in early childhood education (50.00%). Most of the teachers under this profile had more than three years of teaching experience (83.30%). There were relatively few young teachers between 20 to 30 years (16.70%), compared to teachers under other profiles.

Profile 4: The Lowest-level of PCK in early mathematics (n=12, prevailence= 8.10%)

Teachers under profile 3 showed the lowest sensitivity to mathematics during children's play (*M* =1.67). They struggled to think of math questions during children's play (*M*=1.60). Most of these teachers were experienced teachers (85.70%). Teachers in this profile also tended to be older teachers. For example, 22.20% of teachers were between 40-50 years old in this profile, while only 8.70% of teachers were between 40-50 years old in the highest-level of PCK in early mathematics. In this profile, no teacher had a master's degree.

Discussion

The purpose of this study was to identify profiles of Korean preschool teachers by three facets of PCK in early mathematics. With a person-centered approach, the present study adds to the existing literature by explicating inter-relationships among three distinct sets of teacher knowledge in early mathematics. Furthermore, it contributes to a growing body of research by shedding light on how teacher profiles differ by teacher characteristics, such as major, level of education, and years of teaching experiences.

Relationships between three facets of PCK

For the first research question, the evidence from the present study indicated that there was no significant correlation between content knowledge and sensitivity to mathematics during children's play. In a similar context, there was no significant correlation between sensitivity to math during children's play and teachers' ability to pose complex math questions. The findings of the present study are not consistent with a previous study (Oppermann et al., 2016) that found that content knowledge predicts teachers' sensitivity to mathematical concepts. One possible reason for this inconsistency may have been drawn from differences in the study context. While Oppermann and her colleagues (2016) had investigated preschool teachers' content knowledge in Germany, where they have a long history of play-based learning (Wu, 2015), our study examined Korean preschool teachers, who struggle with teaching mathematics during children's play (Lee, 2012). Numerous studies have suggested that teachers may have a different pedagogical stance across nationalities (Ahn, 2015; Choi, 2019; Li et al., 2015). Thus, Korean teachers' content knowledge in teacher-directed math instruction and play may have been disconnected.

Latent profiles of three facets of PCK in early mathematics

The second research question was on identifying latent profiles of teachers by three facets of PCK in early mathematics. One of the significant findings from this study is that there were little variations amongst the four profiles of teachers by the level of content knowledge in mathematics. However, sensitivity to mathematics during children's play and the complexity of math questions was the key determinant that set four distinct groups of teachers. This finding is somewhat consistent with previous studies that showed many Korean preschool teachers have difficulties in facilitating mathematical learning through play (Cho & Park, 2014; Hong, 2010; Lee, 2012). It may be that there were significant variations in teachers' ability to be sensitive to mathematical concepts during children's play because it is a relatively new concept to most Korean preschool teachers (Kim & Park, 2017; Lee, 2017). Hence, experienced teachers may not have much knowledge or training experience for recognizing mathematical concepts during children's play. Or recognizing mathematical concepts during children's play may require teachers with advanced sets of content knowledge. For example, teachers need to be equipped with formative assessment skills to diagnose children's current level of mathematical understanding during play (Haug & Ødegaard, 2015).

Additionally, evidence from the present study indicates that the three facets of PCK in early mathematics are distinct sets of knowledge and skills that are qualitatively different. For example, the findings of the current study show that there is a latent profile of teachers with a low level of subject knowledge in mathematics but with a high level of pedagogical knowledge. Specifically, teachers in Profile 4 were able to pose cognitively challenging questions without being able to recognize the correct mathematical concepts in the given scenario. This finding is inconsistent with previous studies that assume teachers' content knowledge is the prerequisite for teachers' pedagogical knowledge, such as the complexity of math questions (Cross et al., 2009). The finding suggests that it is one thing to recognize relevant mathematical concepts during play situations, and it is another to be able to provide cognitively challenging math questions (Gasteiger, 2014; van Oers, 2010). This finding calls for more intervention research on supporting Korean preschool teachers with three facets of PCK in early mathematics.

Latent profile analysis by teacher characteristics

The third research question pertains to differences in the latent profiles of teachers by their characteristics. The findings from the present study indicate that the teachers in the highest level of PCK profile (profile 2) were likely to major in early childhood education, which suggests the importance of training from preservice education. Studies have indicated that teachers with early childhood education majors tend to have higher expertise because the curriculum is much more rigorous with stringent requirements compared to early childhood care and family majors (Park et al., 2017). This issue stems from a division between early childhood education and child-care systems in South Korea (Park et al., 2017). Therefore, more studies are needed to find specific ways to enhance teachers' PCK in early mathematics for teachers who major in early childhood care and family studies.

Additionally, the level of education level was not related to significant differences across the four profiles of teachers in this study. This study is inconsistent with the previous study that teachers with higher educational levels had higher content knowledge in mathematics (Lee, 2010). One possible reason may be that sensitivity to mathematics during children's play may not be promoted by obtaining a higher educational degree. Interestingly, the result from the current study indicated that the majority of teachers under a high-quality class (i.e., profile 2) were preschool teachers between 20 to 30 years old (65.20%). In contrast, teachers under low-quality class (i.e., profile 4) had the largest proportion of experienced teachers aged between 40 to 50 years old. This finding suggests that years of teaching experience may not promote PCK in early mathematics unless teachers are provided with effective professional development experiences (Pelkowski et al., 2019).

Limitations

Although this study investigated profiles of Korean preschool teachers' PCK with a person-centered approach, it has several limitations. First, the source of this study was mainly drawn from the teacher survey. As the data was collected by an open-ended survey format, teachers had more time to come up with thought-provoking questions compared to actual classroom settings where teachers are distracted by a myriad of demands for classroom instruction and management. Given that studies have reported discrepancies between teacher-reported practices and actual practices in the classroom (Ahn, 2015; Kaymakamoglu, 2018), we recommend future researchers to utilize observational methods. Second, the present study did not examine the association between teachers' PCK in early mathematics and children's math achievement. Thus, we need more evidence on how PCK in early mathematics may be directly or indirectly linked to children's math achievement. Third, there was a discrepancy in the mathematical content areas between two measurements. For example, content knowledge in mathematics assesses five mathematical content areas: number, shape, order, pattern, and comparison. In contrast, sensitivity to mathematics during children's play measures seven content areas: number, operation, shape, space, pattern, measurement, and data analysis. Thus, if we were to compare the inter-relationships between content knowledge in teacher-directed math instruction and teachers' sensitivity to mathematics during children's play, there is a need to develop a comprehensive measurement to assess the multifaceted PCK in early mathematics.

Implications for Practice and Policy

The current study highlights the importance of supporting preschool teachers with a multifaceted PCK in early mathematics beyond the dichotomy between teacher-directed math instruction and play-based learning. The recent development of context-based professional development that enhances everyday moment interaction (Lee et al., 2012; Pelkowski et al., 2019) provides implications for developing comprehensive professional development opportunities in South Korea. For example, many Korean teachers would benefit from watching video clips of exemplary mathematical questions that engage children with high-order thinking (Lee et al., 2012). Opportunities to analyze mathematical interactions in actual classroom practices might help Korean teachers to enhance their PCK in early mathematics (Pelkowski et al., 2019). A dialogical approach that focuses on high-quality teacher-child interaction (Purpura et al., 2017) would promote teachers' ability to pose complex math questions.

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